Nonlocal bright spatial solitons in defocusing Kerr media supported by PT symmetric potentials

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It is studied the bright spatial solitons in nonlocal defocusing Kerr media with parity-time (PT) symmetric potentials. We find that these solitons can exist and be stable over a different range of potential parameters. The influence of the degree of nonlocality on the solitons and the transverse energy flow within the stable solitons are also examined. © 2011 Optical Society of America

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Generally, defocusing Kerr nonlinearity can induce an enhanced beam broadening so that it does not support any localized structures other than vortex and dark solitons, which require background beams [1, 2]. However, the bright solitons, which were thought to exist only for focusing nonlinearity [1], can in fact exist in periodic photonic lattices or waveguide arrays with defocusing nonlinearity [3–5]. This means that the strong transverse periodic refractive index potentials can not only suppress the beam broadening due to diffraction but also overcome the broadening effect due to the defocusing Kerr nonlinearity [1].

The nonlocal nonlinear behavior in a system is related to the change of nonlinear refractive index, which depends not only on the intensity of a local beam but also on the intensity of the surrounding region due to a specific distribution [6]. The nature and extent of nonlocality substantially depend on the materials. Principally, new effects attributed to nonlocality have been studied in thermo-optic media [7], photorefractives [8], and liquid crystals [9, 10].

The definition of PT potentials and their properties were discussed in the past few years [11–13]. The real part of PT symmetric potentials must be a symmetric function of position, while the imaginary component should be antisymmetric. Recently, parity-time symmetric potentials have been introduced into optical field [14–27]. However, thus far all studies focus on bright solitons in self-focusing optical PT symmetric media, and bright spatial solitons in nonlocal Kerr self-defocusing media with a single PT complex potential are never reported.

In this paper, we investigate the bright spatial solitons in nonlocal defocusing Kerr media with parity-time (PT) symmetric potentials. It is found that these self-tapped states can exist and be stable over a different range of potential parameters. In addition, we show the influence of the degree of nonlocality on the solitons and the transverse energy flow within the stable solitons.

In a nonlocal Kerr self-defocusing medium with PT symmetric potentials, the one-dimensional optical beam evolution is governed by the following normalized nonlinear Schrödinger-like equation for q and ϕ, which are respectively the dimensionless light field amplitude and the nonlinear correction to the refractive index [14–25],

$$i\frac{\partial q}{\partial z} + \frac{\partial^2 q}{\partial x^2} + [V(x) + iW(x)]q - \phi q = 0, \quad (1a)$$

$$\alpha^2 \frac{\partial^2 \phi}{\partial x^2} - \phi + |q|^2 = 0. \quad (1b)$$

where z is the propagation distance, $V(x)$ and $W(x)$ are the real and the imaginary components of the complex PT symmetric potential, respectively. $V(x)$ is an even function and $W(x)$ is odd. Physically, $V(x)$ is associated with index guiding while $W(x)$ represents the gain/loss distribution of the optical potential. We assume that the depth of the refractive index modulation is small compared to the unperturbed index. The nonlocality of the materials is supposed to be ruled with an exponential response function $g(x) = 1/(2\alpha^{1/2}) \exp(-|x|/\alpha^{1/2})$ (as in liquid crystals), where α is the degree of the nonlocality. We are going to search for a stationary soliton solution of Eq. (1) in the form of $q(x, z) = u(x)e^{ibz}$, where u is a complex function and b is the propagation constant of spatial solitons. In this case u satisfies

$$i\frac{\partial u}{\partial z} + \frac{\partial^2 u}{\partial x^2} + [V(x) + iW(x)]u - \phi u = 0, \quad (2a)$$

$$\alpha^2 \frac{\partial^2 \phi}{\partial x^2} - \phi + |u|^2 = 0. \quad (2b)$$

Here, we assume a Scarff II potential shown in Fig. 1(a) where $V(x) = V_0 \text{sech}(x)^2$ and $W(x) = W_0 \text{sech}(x)\tanh(x)$, with V_0 and W_0 being the amplitudes of the real and imaginary part [12, 14, 15]. Although the PT symmetric potential has crossed the phase transition point, the solitons still exist because the amplitude of the refractive index distribution can be altered by the beam

itself through the optical nonlocal nonlinearity. The PT symmetric will remain broken if it cannot be nonlocal nonlinearly restored [14, 15, 17, 18].

To check the stability of the solitons with the method of linear stability analysis, we assume $q(x) = u(x)e^{ibz} + \epsilon[F(x)e^{i\delta z} + G^*(x)e^{-i\delta^* z}]e^{ibz}$, where $\epsilon \ll 1$, F and G are the perturbation eigenfunctions, and δ is the growth rate of the perturbation. By linearizing Eq. (1), we gain [14]

$$\delta F = \frac{\partial^2 F}{\partial x^2} + (V + iW)F + nF - bF + u\Delta n, \quad (3a)$$

$$\delta G = -\frac{\partial^2 G}{\partial x^2} + (-V + iW)G - nG + bG - u^*\Delta n. \quad (3b)$$

where $n = -\int_{-\infty}^{+\infty} g(x - \lambda)|u(\lambda)|^2 d\lambda$, and $\Delta n = -\int_{-\infty}^{+\infty} g(x - \lambda)[G(\lambda)u(\lambda) + F(\lambda)u^*(\lambda)]d\lambda$. The bright spatial soltions are linearly unstable when δ has an imaginary component, on the contrary, they are stable when δ is real.
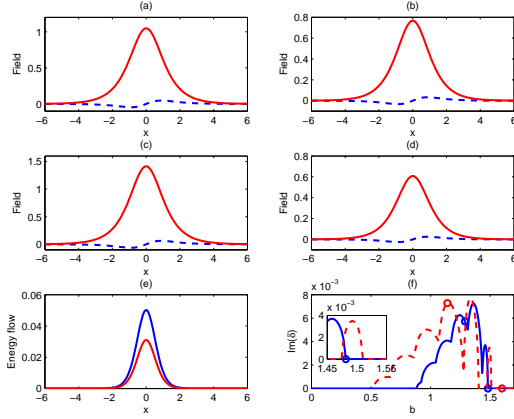

Fig. 1. (color online) (a)-(d) The real (solid red curve) and imaginary components (dash blue curve) of the soliton solutions: (a) $b = 1.288$, $\alpha = 2$, (b) $b = 1.481$, $\alpha = 2$, (c) $b = 1.141$, $\alpha = 5$, and (d) $b = 1.599$, $\alpha = 5$. (e) Transverse energy flow S of solitons(blue and red curves represent the solitons in Figs. 1(b) and (d), respectively.) (f) Im(δ) versus b at $\alpha = 2$ (solid blue curve) and at $\alpha = 5$ (dash red curve), where the points marked with circles correspond to the solitons in Figs. 1(a)-(d), and the inset depicts the curves from $b = 1.45$ to $b = 1.55$. The potential parameters are $V_0 = 3.01$ and $W_0 = 0.3$.

In order to gain the solitons solutions, we numerically solve Eq. (2) using spectral renormalization method [28]. We find a family of the simplest ground-state nonlocal bright solitons with PT symmetric potentials. To illustrate the properties of the solitons, we vary b, α, W_0, and V_0. First, the typical cases of the solitons with different b and α are shown in Fig. 1(a)-(d), where $V_0 = 3.01$ and $W_0 = 0.3$. Here the fields of unstable and stable soliton are respectively depicted. Evidently, the change of b and α can influence the solutions. To shed more light on the
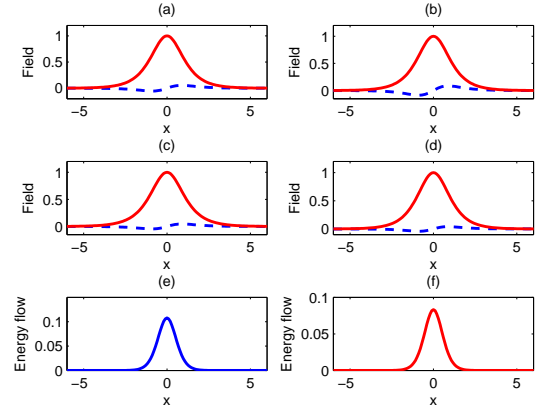

Fig. 2. (color online) (a)-(d) The real (solid red curve) and imaginary components (dash blue curve) of the soliton solutions: (a) $W_0 = 0.38$, $V_0 = 3.01$, (b) $W_0 = 0.60$, $V_0 = 3.01$, (c) $V_0 = 2.8$, $W_0 = 0.3$, and (d) $V_0 = 3.15$, $W_0 = 0.3$. (e) and (f) Transverse energy flow S of solitons in Figs. 1(a) and (d), respectively. The other parameter is $\alpha = 5$.

properties of the stable solitons, we study the parameter $S = (i/2)(uu_x^* - u^*u_x)$, which is associated with the transverse power flow density or Poynting vector across the beam [14]. The transverse energy flow density S is shown in Fig. 1(e), and the fact that S is positive everywhere implies that the energy always flows in one direction, i.e., from the gain region toward the loss region [14]. It is shown in Fig. 1(f) how Im(δ), the imaginary component of δ, varies versus b. Since Im(δ) is not always equal to zero, the solitons will not always be stable. One can see that, when α is different, the stable regions of the solitons are different too, which means the degree of nonlocality can influence the stability of solitons.

Next, we set $\alpha = 5$ and then vary W_0 and V_0. In Figs. 2(a)-(d), the bright solitons in different conditions are shown. Figs. 2(e) and (f) depict the transverse energy flow S of the solitons in Figs. 1(a) and (d), respectively. Similarly, one can find that S is positive everywhere, which also means that the energy always flows in one direction. When $V_0 = 3.01$, we gain the relations of b and Im(δ) with W_0 shown in Figs. 3(a) and (b), respectively. Fig. 3(a) shows that the bright solitons can exist when $0 < W_0 < 3.13$. Thus for a fixed value of V_0, there exist a threshold for the imaginary amplitude W_0. Above this threshold ($W_{0upp} = 3.13$), a phase transition occurs and the spectrum enters the complex domain [14]. Moreover, Figs. 3(c) and (d) show how b and Im(δ) vary with V_0 when $W_0 = 0.3$. In Fig. 3(c), one finds that the bright solitons can exist when $V_0 > 1.03$, and thus, for given W_0, this effective potential nonlinearity will shift the PT V_0 threshold ($V_{0low} = 1.03$) and in turn allow nonlinear eigenmodes with real eigenvalues to exist. In Fig. 3(b) and (d), Im(δ) is not always equal to zero, so the solitons are not always stable. To examine linear stability results,
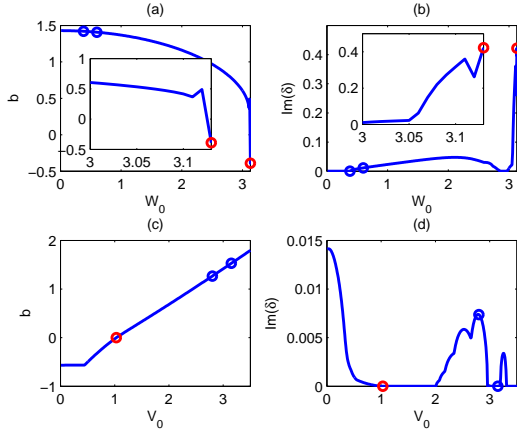
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Fig. 3. (a) The propagation constant b versus W_0 at $V_0 = 3.01$. (b) Im(δ) versus W_0 at $V_0 = 3.01$. (c) The propagation constant b versus V_0 at $W_0 = 0.3$. (d) Im(δ) versus V_0 at $W_0 = 0.3$. Points marked with blue circles correspond to the solitons in Figs. 1(a)-(d), respectively. Points marked with red circles correspond to the threshold $W_{0upp} = 3.13$ for the existence of the bright solitons where b becomes negative in (a) and (b). Points marked with red circles correspond to the threshold $V_{0low} = 1.03$ for the existence of the bright solitons where b becomes negative in (c) and (d). The insets depict the curves from $W_0 = 3$ to $W_0 = 3.13$ in (a) and (b). The other parameter is $\alpha = 5$.

we further simulate the propagation of beams with random noise in the different conditions corresponding to Figs. 1(a)-(d) and Figs. 2(a)-(d), see Figs. 4.

To summary, the bright spatial solitons in nonlocal defocusing Kerr media with PT symmetric potentials are studied. The existence, stability, and propagation dynamics of such solitons are discussed in detail.

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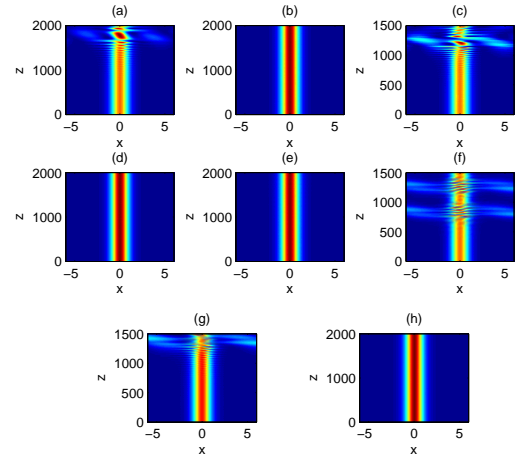
Fig. 4. (a)-(d) Simulated propagation of the solitons with 1% random noise corresponding to Fig. 1(a)-(d). (e)-(h) Simulated propagation of the solitons with 1% random noise corresponding to Fig. 2(a)-(d).

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